Day 2

Moving a load

Topics of discussion:

- Torque and effective tension requirements
- Fixed-belt arrangement
- Conveyors

Timing belts in linear positioning

Day 1 of the series "Timing belts in linear positioning" discussed the geometry of timing belt systems — belt and pulley pitch, variations in belt and pulley contact with different timing belt styles, and designation of belt length with respect to pulley size and center distance. With that established, we can now look at the dynamics of moving a load.

Here we focus on belt forces required to move a load linearly and on breaking down all the components of motion resistance. Some attention will also be given to rotary motion and power transmission drives, both to set up the analysis of linear systems and to highlight the differences.

Torque and effective tension requirements

Rotary motion and power transmission systems (*figure 1*) have driving and driven *shafts* and *pulleys*. A linear system, on the other hand, drives a *slider* or positioning *platform* attached to the belt. In a typical linear positioning arrangement (*figure 2*) the belt is stretched around a driving and an idler sprocket, which are usually equal diameters.

A loaded belt drive has different belt tensions at the tight and slack sides. (The tight side is where the belt is pulled toward the driving pulley.) The difference in tension is the effective tension T_e and is given by the equation:

$$T_e = T_1 - T_2$$

where T_1 and T_2 are the tight and slack side tensions, respectively. Effective tension T_e represents the net force transmitted from the driving pulley to belt.

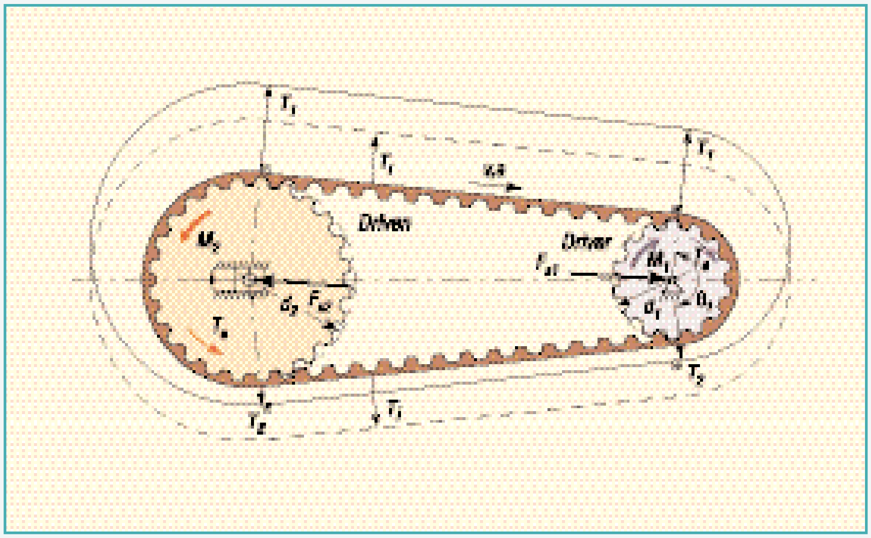

***Figure 1* — power transmission linkage**

In a power transmission or rotary positioning arrangement, load movement, or work, is accomplished by turning the output shaft and pulley — usually with a speed reduction and torque increase.

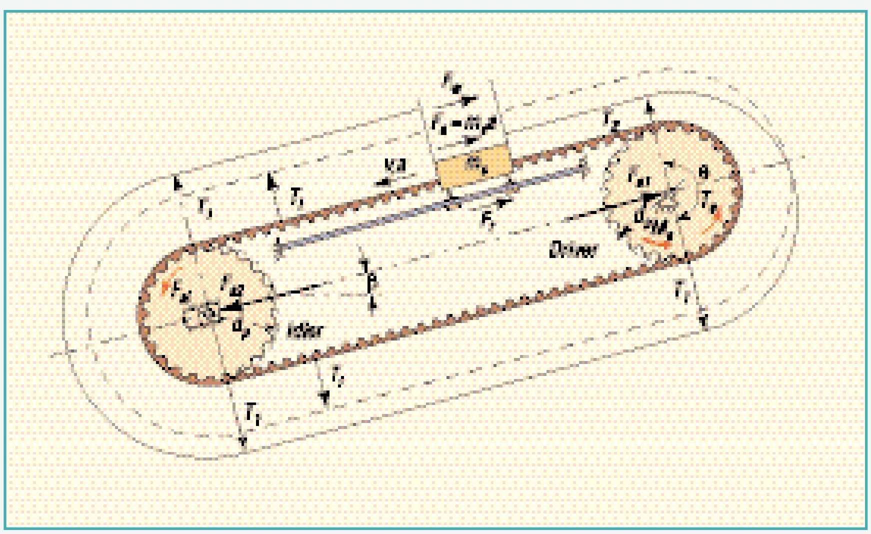

***Figure 2* — linear actuator**

A typical linear positioning system offsets the driving pulley with an idler pulley, usually of equal diameter. The load is linked to the belt and moved parallel to the span between the pulleys. This system contains a platform or slider that moves on linear bearings.

Driving torque M (or M_1 in the power transmission setup in *figure 1*) is given by:

$$M = T_e \left(\frac{d}{2} \right)$$

where d (d_1 in *figure 1*) is the driving pulley pitch diameter.

Again, effective tension is the actual working force, the force that overcomes resistance to belt motion. All forces acting on the belt must be assessed, as they contribute to the effective tension and hence to the torque required at the driving pulley.

In power transmission, motion resistance develops at the driven pulley. Effective tension amounts to the force transmitted from the driving to driven pulley. The torque required at the driving axis can be written as:

$$M_1 = T_e \left(\frac{d_1}{2} \right) = \frac{M_2}{\eta} \left(\frac{d_1}{d_2} \right) = \frac{P_2 d_1}{\omega_2 \eta d_2} = \frac{P_2}{\omega_1 \eta}$$

where M_1 is, again, the driving torque; M_2 is the required torque at the driven pulley; P_2 is the required power at the driven pulley; ω_1 and ω_2 are, respectively, the angular speeds of the driving and driven pulleys; d_1 and d_2 are, respectively, the pitch diameters of the driving and driven pulley; and η is the belt drive efficiency, usually around 0.94 to 0.96.

The angular speeds (rad/sec) of driving and driven pulley are related according to the formula:

$$\omega_2 = \omega_1 \left(\frac{d_1}{d_2} \right)$$

And ω_1 and ω_2, being rad/sec, are easily converted to n_1 and n_2 which are rpm:

$$\omega_{1,2} = \frac{\pi n_{1,2}}{30}$$

Linear positioning requires different notation and equations than power transmission. With a linear actuator like the one in figure 2, the main load acts at the positioning platform or slider bed. The total driven load consists of the acceleration force Fa (due to linear acceleration of the platform); friction force Ff developed at the linear bearings or any surfaces supporting the load; the external force or work load Fw; the actuated mass's gravitational force component Fg acting parallel to the belt in an inclined drive; inertial force Fab required to accelerate the belt; and inertial force F_{ai} to accelerate the idler pulley. The required effective tension is the sum of all these terms:

$$T_e = F_a + F_f + F_w + F_g + F_{ab} + F_{at}$$

Each of these individual T_e components can be further broken down:

$$F_a = m_s a$$

where m_s is the actuated mass and a is the linear acceleration of the mass.

$$F_f = \mu_r m_s g \cos\beta + F_{fi}$$

where μ_r is the linear bearing's dynamic friction coefficient (you can usually get it from the bearing manufacturer); F_{fi} accounts for all linear resistance independent of load — seal drag, preload resistance, drag from lubricant, and so on; β is the load path's angle of inclination; and g is gravity.

$$F_g = m_s g \sin\beta$$

$$F_{ab} = \frac{w_b L b a}{g}$$

where L is belt length, b is belt width, and w_b is the belt's specific weight.

$$F_{ai} = \frac{2 J_i \alpha}{d} = \frac{a m_i}{2} \left(1 + \frac{d_b^2}{d^2} \right)$$

where J_i is the idler pulley inertia, α is the idler's angular acceleration, m_i is the idler mass, d is the idler diameter, and d_b (where applicable) is the idler bore diameter.

Fixed-belt arrangement

The typical linear positioning arrangement in *figure 2* doesn't cover it all, however. A variation is shown generically in *figure 3*. Here, the belt is not the moving element. Instead, the driving axis translates together with the loaded platform and two idler rolls that roll along the back of the belt (the smooth side of a single-

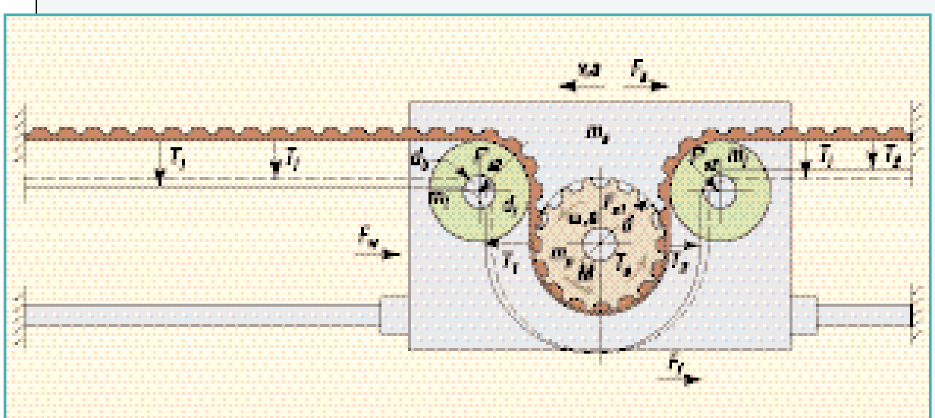

***Figure 3* — moving on a fixed belt**

In this linear positioning system, the driving axis, idler rollers, and loaded platform proceed as a unit along a length of belt that's held fast at both ends. Belt and drive sprocket impel the load while linear bearings support it.

sided timing belt). The belt is clamped in place at the ends, and the drive sprocket engages the belt and progresses down the line.

In such a system, the effective tension is given by:

$$T_e = F_a + F_f + F_w + F_g + 2F_{ai}$$

Notice that F_{ai} (the force required to accelerate the idler) is multiplied by two to account for the pair of idler rolls. And, with a stationary belt, the force F_{ab} is no longer necessary.

Furthermore, we have to revise our definitions of the individual terms that make up T_e:

$$F_a = a\left(m_s + m_p + 2m_i\right)$$

where m_s is the slider or platform mass, m_p is the driver pulley mass, m_i is the idler rollers' combined mass, and a is the translational acceleration of the platform.

$$F_f = \mu_r\left(m_s + m_p + 2m_i\right)g\cos\beta + F_{fi}$$

$$F_g = \left(m_s + m_p + 2m_i\right)g\sin\beta$$

$$F_{ai} = \frac{4J_i\alpha}{d} = m_i a\left(1 + \frac{d_b^2}{d_i^2}\right)$$

Remember, here the idler rollers' diameters are often different than that of the driving pulley; therefore, quantify J_i as the inertia *reflected* to the driving pulley.

Conveyors

Now let's take a close look at the mechanics of an inclined conveyor drive, as shown in *Figure 6*. Here, effective tension must account for two primary resistive forces, frictional and gravitational. Both act parallel to the belt and load path. The friction force F_f is given by:

$$F_f = \mu\sum_{k=1}^{n_c} N_{(k)} = \mu\cos\beta\sum_{k=1}^{n_c} W_{(k)}$$

Where μ is the friction coefficient between the belt and the stationary slider bed that supports from underneath (recall that in the actuator in *figure 2*, F_f is due to friction in the linear bearings supporting the load platform); $N_{(k)}$ is a package's weight component acting normal to the belt surface; $W_{(k)}$ is the total weight of a package; n_c is the number of packages on the conveyor; index k designates the kth package on the belt; and β is the incline angle.

With granular materials — a continuously distributed medium rather than discrete packages — we identify friction force by the equation:

$$F_f = \mu w_m L_m \cos\beta$$

where w_m is the material weight distribution per unit of length along the conveyor path, and L_m is the total conveyed length.

In *figure 4*, there is another factor present: material accumulation. The packages gather and come to rest toward the removal end of the conveyor path. As the smooth (upper) side of the belt slides under the now-stationary packages, it causes an additional friction force F_{fa}:

$$F_{fa} = \left(\mu + \mu_1\right)\sum_{k=1}^{n_a} N_{(k)} = \left(\mu + \mu_1\right)\cos\beta\sum_{k=1}^{n_a} W_{(k)}$$

where n_a is the number of accu-

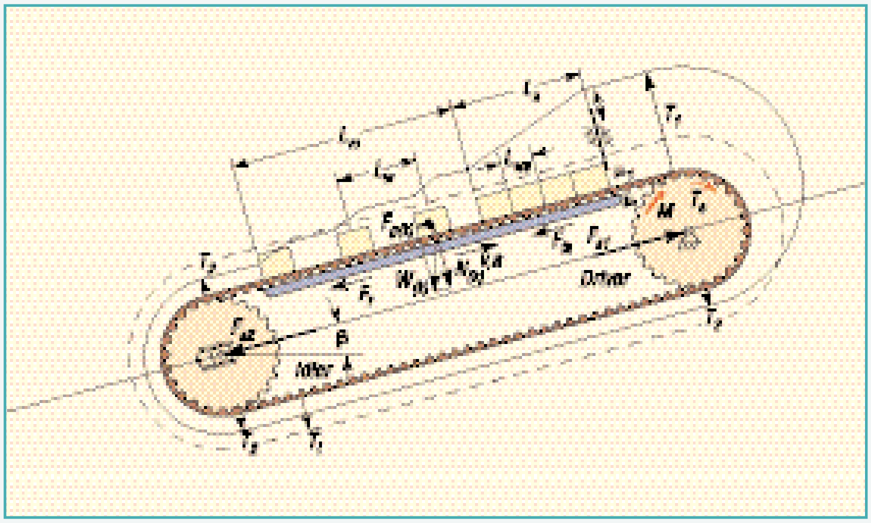

***Figure 4* — inclined conveyor with accumulation**

Effective belt tension must overcome gravity, friction between the support bed and underside of the belt, and friction between the top (smooth) of the belt and the accumulated packages. Note the subsequent increase in tight-side belt tension as packages (and friction forces) stack.

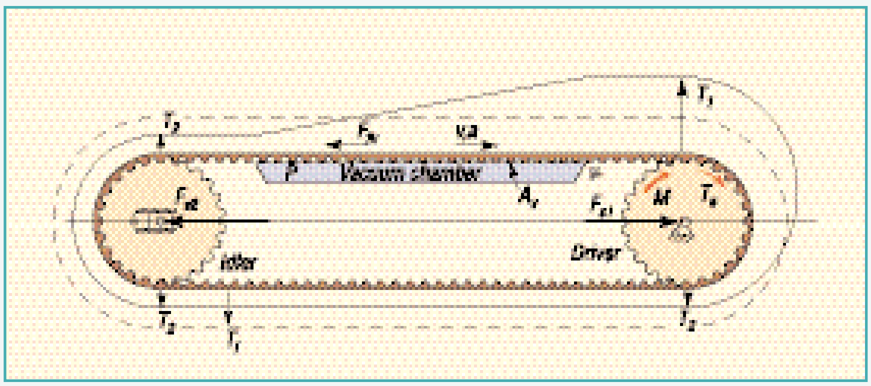

***Figure 5* — vacuum conveyor**

A vacuum-assist slider (support) bed pulls the belt flush against it. The friction thereby created is the primary resistance to motion. The linear increase in tight side tension is attributed to uniform pressure along the length of the slider bed.

mulated packages and μ_1 is the friction coefficient between the belt and accumulated material. This equation can also be written as:

$$F_{fa} = (\mu + \mu_1) w_{ma} L_a \cos\beta$$

where w_{ma} is the accumulated packages' weight distribution per unit of length and L_a is the length of accumulation.

Now, to get the required effective tension we still need the gravitational load, which can be written as:

$$F_g = \sin\beta \sum_{k=1}^{n_c+n_a} W(k)$$

The above equation can be rewritten:

$$F_g = (w_m L_m + w_{ma} L_a)\sin\beta$$

In a vacuum conveyor, illustrated in *figure 5*, the weight of the conveyed elements is usually negligible. The main source of resistance and, hence, the main component of effective tension is the friction force F_{fv} caused by the vacuum between belt and slider bed. It is given by:

$$F_{fv} = \mu P A_v$$

where μ is the friction coefficient between belt and bed, P is the magnitude of vacuum pressure relative to atmospheric pressure, and A_v is the total area of vacuum openings in the stationary slider bed. With a uniform pressure distribution, there is a linear increase in tight side tension in the direction of belt motion, as can be seen in *figure 5*. ●

Information for Course Audit this month was provided by Krzysztof Kras, engineering manager, Mectrol Corp., Salem, N.H.

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